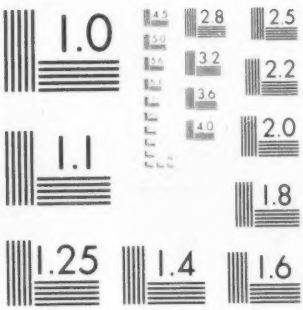

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ABSTRACT

Investigated was the effect of format variables that describe mathematical word problems in relation to problem-solving difficulty. Changes made by sixth- and eighth-grade students when asked to rewrite problems in a more adequate or easily understandable format were analyzed. Thirty-five sixth and eighth graders enrolled in a Mentally Gifted Minors program in a large affluent middle school participated in this study. Students were asked to rewrite the word problems individually in a quiet classroom setting. Each student completed the task within a 15- to 50-minute time period depending upon the number and type of problems to be rewritten. The analysis of the rewritten problems shows that the format changes made by students can be classified into twelve categories. Examples of the type of format changes made by some students are also included. (HM)

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An Analysis of the Changes Made by Sixth and Eighth Grade Students When Rewriting Arithmetic Word Problems

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SE 027 788

An Analysis of the Changes Made by Sixth and Eighth Grade Students When Rewriting Arithmetic Word Problems

1. INTRODUCTION

Finding methods to increase problem-solving ability, especially in the area of arithmetic word problems, has long been an area of critical and needed research in mathematics education. Fortunately, as more research in this area is carried out, the more we as teachers can learn about how to approach the awesome problem of teaching problem-solving techniques and skills to our students. Prior studies of problem-solving ability range from straight-forward comparisons of group performance to intricate factor or problem analysis. It is the purpose of this study to investigate the effect of format variables that describe mathematical word problems in relation to problem-solving difficulty. In this study students were asked to rewrite textbook word problems to make them easier. An analysis of the rewritten problems shows that common format variables, such as the use of diagrams, the grammatical and linguistic complexity of word problems, symbol to word ratio and the use of extraneous information, are perceived by the children as related to the difficulty of the problem.

In essence, this study attempts a "naturalistic" manipulation of what has before been artifically manipulated by researchers and curriculum writers by letting the student express his intuitive "gut level" feeling about word problems in a free form style.

2. SUBJECTS

The experimental subjects were sixth and eighth graders enrolled in a Mentally

Gifted Minors program in a large affluent middle school; all the subjects were white or Asian middle class students. Although the students in the experimental group were identified as gifted by the Stanford Binet Test of Intelligence and their I.Q. score ranged from 132 and above, in several cases their mathematics ability was in the average range and many students were not significantly above grade level in arithmetic computational skills.

3. CURRICULUM

The word problems used in this study were randomly selected from a 700 word problem sample used in a earlier study (Searle, Lorton, Suppes 1974) which identified structural variables that affected performance of students on arithmetic word problems and assessed the usefulness of these variables as predictors of success. The predicted probability correct for the problems in the original study mentioned above ranged from .01 to .89, based on a sample of disadvantaged fourth grade students. The fifteen problems selected for this study covered most of the difficulty range and were of different types as defined by the earlier study. (The problems shown in the Appendix are presented in order of difficulty and are numbered from the least difficult to the most difficult).

4. DESIGN

Design of Study

A pilot study was conducted in this experiment to examine the feasibility of the procedure: would students indeed be able to rewrite common mathematical textbook problems? Would it be possible to analyze the types of changes being made? It was found that students were able to rewrite common math word problems, and also that

every student who participated in the study made at least three format changes per problem.

After finding that each student could rewrite common math textbook word problems and were capable of handling this task, a total of ninety problems were given to thirty-five sixth and eighth grade students for revision. Since it was important that there be an adequate overlap in the word problems that were rewritten, each of the fifteen problems analyzed in the study were rewritten by six different students (three sixth and three eighth), so that a comparison or analysis by problem type could be made. Table I shows a breakdown of the format changes by problem. Each of the thirty-five children who participated in the study rewrote from one to six problems.

Insert Table I here

An analysis of the rewrites identified twelve specific format changes that occurred most frequently. As a result, a checklist to analyze the problem rewrites consisted of the twelve format changes found to be most consistently manipulated in the study. Based on the format changes in the checklist, each problem was analyzed separately and a comparison of sixth and eighth grade changes was made as well as a composite analysis of the total types of format changes made in all ninety problems. Also upon completion of the task, each student was interviewed individually and was asked to describe the rationale behind each change made in their rewrites.

5. PROCEDURE

All thirty-five students who were asked to rewrite the word problems did so individually in a quiet classroom setting. Each student completed the task within a fifteen to fifty minute time period depending upon the number and type of problems

to be rewritten. In all cases however, the eighth graders completed the task in less time than the sixth graders.

The instructions given to all students prior to the experiment were written and they were as follows:

INSTRUCTIONS:

> Often when constructing word problems, it is difficult to write them in a way that is clear for kids to understand. Take the following problems and imagine that they will be given to kids less knowledgeable than yourself in mathematics. Rewrite these problems (you may add and delete) so that they will become simpler and easier for the next group to solve.

USE YOU IMAGINATION!

No further information, such as examples of what could be done to simplify a problem, was given at that time in order not to influence the results. There was also no time period or constraint given in the experiment and each student was given a minimum of one full class period consisting of fifty minutes to complete the task.

The following day after all word problem revisions were assembled, an individual interview was conducted and an explanation was given by each child for their format change. An in depth individual discussion was also carried on with each child to find out why they felt one format type appeared easier than another. These interviews not only proved to be interesting, but also added a great deal of information to the study.

5. RESULTS

The first analysis of the "rewrites" shows that the changes made by children can be classified into twelve categories. These changes include:

1. Simplified vocabulary
2. Made a long sentence a series of shorter ones
3. Added a diagram
4. Removed extraneous information
5. Took out "if"
6. Took out "about"
7. Made the question sentence separate
8. Added information or changed story line
9. Changed order of numerical information
10. Gave clue
11. Changed verb to present tense
12. Changed pronoun to proper noun

Description of Format Changes: The classification "Simplified vocabulary" includes changes in mathematical terms and also changes that do not involve mathematical language as shown in Tables II and III. A majority of the vocabulary changes were made in seven of the fifteen different problems.

Insert Tables II and III here

When making a long sentence into a series of shorter ones, most students shortened compound or complex sentences. The mean sentence length after revision was consistently shorter than the mean sentence length in the orginal. An example of a sentence length change in the study was "Scott filled 26 boxes with 6 books in each box" to "Scott had 26 boxes. He put 6 books in each box." This type of format change was very common in almost all of the fifteen problems.

When adding a diagram, it appeared that certain problem types lended themselves more often to a diagram or pictoral representation. The majority of these problems were found to be the most difficult for students to solve in the previously mentioned Stanford study and none of them contained diagrams in their original format before the rewrites. The majority of the problems in which a diagram was added involved geometry, fractions, percents, and the concept of distance. An example of some of these problems with added diagrams can be in the Appendix.

When removing extraneous information it was found that the students removed information, not suprisingly, from the problems with the most verbage. The mean length of problems from which information was removed was 31.8 words. No information was removed from any sentence under twelve words and this was only done by one student. Most of the words that were removed were either prepositions, prepositional phrases or subordinate conjuctions (i.e. If). In some cases a complete sentence was omitted. Table IV lists several of the omitted words for both grades six and eight.

Insert Table IV here

One of the most interesting findings in this study involved removing the words if and about from common textbook word problems. The word about for example was removed in one hundred percent of the cases. When asked why these words were removed, most students alluded to the fact that they were too abstract or were not concrete or precise enough. This may have to do with the developmental level of the students in the study since all of the subjects were in the eighth grade or below.

When making the questions sentence a separate sentence, all students placed the question at the end of the problem. Many of the students actually removed the question from the body of the problem by moving it down a few spaces or indenting it. In very few cases did the student actually leave the question in body of the problem and in almost no case was the question left as part of a compound sentence.

Students tended to add information or change the story line of a word problem when it appeared too bare to incomplete. An example of the type of format change made by an eighth grade girl is as follows:

Actual Problem: 4. 356 is how much greater than 291?

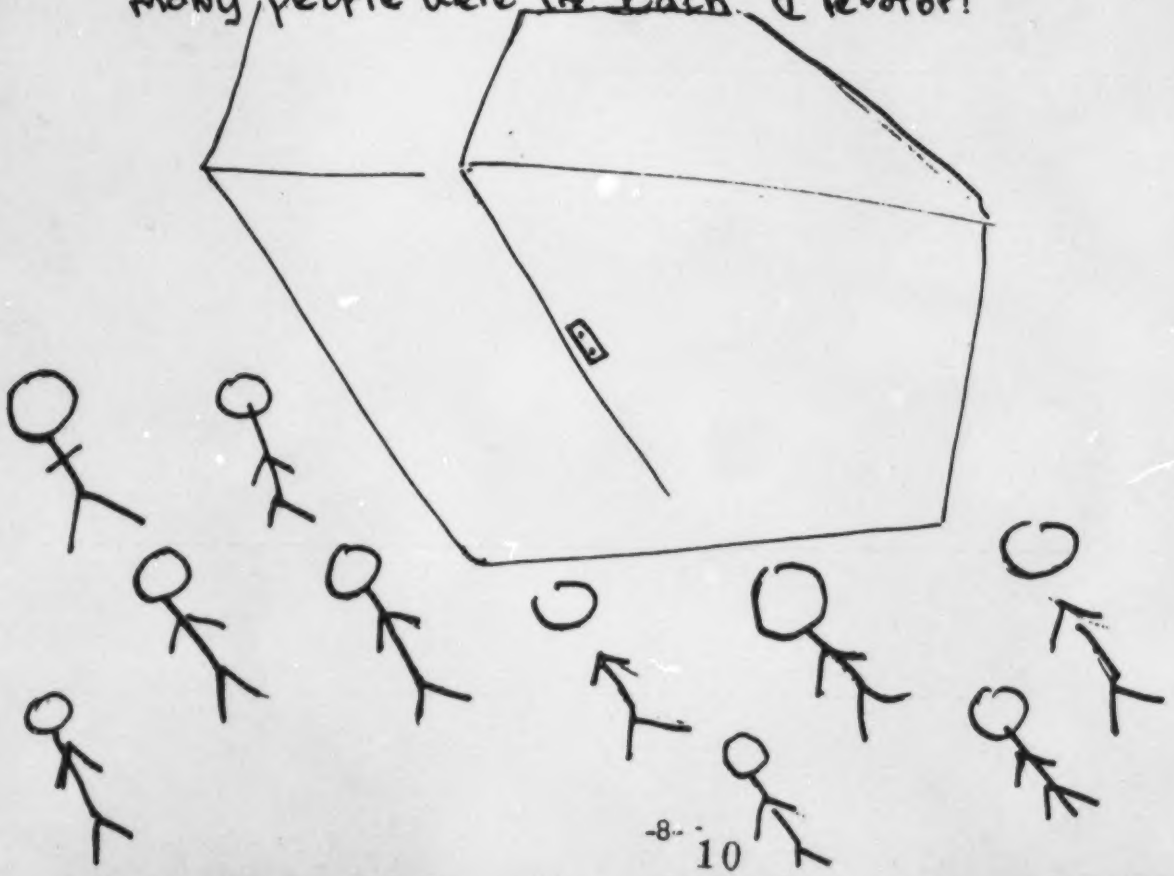
Student Rewrite:

John has 356 jellybeans.
Peter has 291 jellybeans
How many more jellybeans does John have than Pete?

Other students tended to change the storyline of a problem when it appeared irrelevant or confusing to them. One of the most interesting changes here was made a sixth grade boy:

9. A group of 56 people was separated into three smaller groups. Two the the smaller groups had the same number of people. The other smaller group had 10 people. How many people did each of the first two small groups have?

A group of 56 people were waiting to go into two large elevators. When the elevator doors opened most of the people went in them but some couldn't fit. The two elevators had the same number of people in them. There were ten people left out of the elevators. How many people were in each elevator?



The most sophisticated format change made by the students in this study involved changing the order of the numerical presentation. Both sixth and eighth graders made this change indicating that they recognized order as a factor contributing to problem difficulty. This change was made at least twice in twelve out of the fifteen problems used in the study. The following is an example of this type of format change made by a eighth grade boy:

6. The Kant Family has driven 270 miles since they started their trip. The whole trip is 583 miles long. How many miles do they have left to go?

The Kant family is going on a 583 mile long trip. They have driven 270 miles since they started. How many miles do they have left to drive?

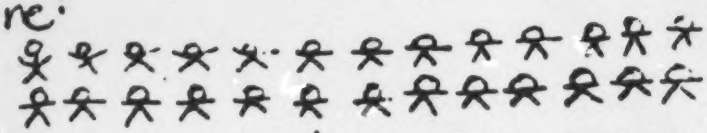
A few students added a clue to seven of the fifteen problem types that were rewritten. Students also changed verbs to the present tense. A few students changed pronouns to proper nouns. An example of a problem in which a student added a clue can be seen in this unique and clever revision by a sixth grade boy:

Actual Problem: Susies class took a field trip to the Zoo. Each student brought $2 to cover lunch, admission to the Zoo, and souvenirs. Miss Rowe collected all the money. She said that lunch cost three-fourths of all the money collected. If there were 26 people in all, how much did the group spend for lunch?

Student Revision:

In Susie's class there are 26 kids. They went on a field trip to the zoo. The kids had to pay $2 each. Lunch costed three-quarters of the money How much did lunch cost for the whole class?

Imagine:



SUSIE'S CLASS (26 KIDS)
EVERY KID PAYS $2.00
THREE-QUARTERS OF ALL THE CLASS' MONEY PAYS FOR LUNCH.
HOW MUCH IS SPENT (TOTAL) FOR LUNCH?

Frequency of Changes:

Some of the format changes were made by one grade more consistently than the other as shown in Table V.

Insert Table V here

For example, the vocabulary was simplified in 71% of the ninety problems that were involved in the study; 40% of the changes were made by sixth graders and 31% were made by eighth graders.

For the format variables, Making a Long Sentence Shorter, Adding a Diagram and Removing Extraneous Information, the percentages were 34% to 37%, 9% to 2% and 23% to 17% for sixth and eighth graders respectively. 47% of the students removed the word "if" from story problems and 30% of them were sixth graders while remaining 17% were eighth graders. Conversly, however, of the 100% of the students who removed the word about in the story problems, 50% of them were in eighth grade. A larger percentage of sixth graders also tended to make the Question Sentence Separate, Add Information to the Story Line and Change the Verb Tense of a Story Problem, while a larger percentage of eighth graders changed the Order of the Numerical Information in a Problem and Changed a Pronoun or Noun to a Proper Noun.

Results of Student Interviews

When students were asked why they made a particular format or structual change in a problem, they gave answers like, "I like to build a story in my mind when I read a word problem -- sort of like I'm making a movie -- that's why I like to know right away what the problem is about. I also like the numbers in the problem to be in the 'right' order." Another sixth grade boy remarked "Yes, I like it when I'm not tricked or when I'm asked a surprise question. For example -- here's a story of Goldielocks and The Three Bears. There is a big papa bear, a mama bear and a baby bear. They

all lived in a small house in the woods. Oh by the way how many bears are there? See what I mean? I hate over-the-shoulder approach." Upon further probing it appeared that this student was saying that he liked a form of an advance organizer, a preference that was expressed by several of the other students in both grades. Other students did not like the words if and about in word problems because they appeared to vague and not concrete enough; they remarked that too much difficult vocabulary or extra words can make a problem confusing. A number of students also expressed the concern that many of the story lines found in word problems in most math textbooks were irrelevant and unrelated to them. Consequently they frequently changed the setting or context of the problem entirely. Finally many students also expressed the need to have fewer compound sentences or, as one girl put it, "one idea per sentence please." Some students also wrote the problems in the present tense, trying to put the reader into the setting. For example, "you and your best friend are going to the store. . . . ." Some of the common word changes can be found in Tables II and III; some of the extraneous words ommitted from problems are presented in Table IV.

7. DISCUSSION

It has been shown that twelve format changes were consistently made by students when rewriting textbook word problems. Although the students in this study differed in outside personal interests and general mathematics ability, several common threads did seem to run through most word problem revisions. The finding of the similarity between children is indeed interesting and may prove significant in the future to both curriculum writers as well as elementary classroom mathematics teachers. Further questions to be raised as a result of analyzing these problem rewrites include such things as; Do sixth and eighth grade students write problems that are more easily understood by their peers? (A simple pre/post test or post test or post test only design with two equivalent groups is necessary to answer this question). If so, what

happens to the problems in terms of the actual format changes that have been made? Another interesting question is, does rewriting problems help those who are revising the problem to become better or more observant problem solvers in the future? All of these are questions that need to be answered. It has already been shown that certain format variables are consistently manipulated by sixth and eighth grade students when asked to rewrite problems in a more adequate or easily understandable format. However, the results obtained so far in this investigation are far from exhaustive and further study should prove rewarding.

TABLE 1
Number of Students Making Each Format Change

Change	Problem Number														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Simplified Vocabulary	6	5	5	3	6	3	6	0	6	5	6	4	4	2	3
Shortened Sentence Length	4	4	4	3	4	4	4	4	6	4	4	5	4	4	6
Diagram	0	1	na	1	na	1	0	0	0	0	0	0	2	1	2
Removed Extraneous Information	3	6	0	0	1	3	5	0	2	0	4	1	2	5	4
If	2	5	na	na	na	na	na	3	na	na	na	na	na	2	2
About	na	na	na	na	na	na	na	na	na	na	na	6	na	na	na
Made Question Sentence Separate	3	4	2	na	na	na	1	5	na	na	2	na	na	4	4
Added Information to Story Line	0	2	1	2	1	4	2	1	3	1	1	1	3	0	1
Changed Order of Numerical Presentation	0	0	3	2	4	5	3	4	0	2	4	2	1	3	4
Gave Clue	3	0	1	1	2	0	0	0	0	0	1	1	0	2	0
Verb to Present Tense	na	na	na	na	na	2	1	na	2	0	0	0	0	0	1
Pronoun/Noun to Proper Noun	na	na	na	na	na	na	3	0	na	2	na	na	na	na	na

Notes: n=6 for each problem; na means not applicable. Problems are presented in ascending order of difficulty.

TABLE II

Analysis of Simplified Vocabulary - Grade 6

Grade	Problem	From	To
6	1	Containing	With
6	1	Having Been Detected	Have Been Found
6	1	Life Bearing Planets	Planets with Life
6	3	What Number Results	When Adding
6	3	What Number Results	How Many Were There
6	4	How Much Greater Than	How Many More
6	4	How Much Greater Than	Subtract
6	5	Product	Multiply
6	5	Twice	Two
6	5	Twice The Product	Two Times as Much
6	11	On The Average	How Many in Each
6	11	Visited	Went

TABLE III

I. Analysis of Simplified Vocabulary - Grade 8

Grade	Problem	From	To
8	1	Average	Every
8	7	What part of	How much
8	7	Left undone	To finish
8	9	Successive	Each
8	9	Gained	They made
8	9	Successive	Each
8	9	Extra	Left over
8	10	Extra	Left over
8	12	Shared	Divided

TABLE IV

Analysis of Extraneous
Information (Grades Sixth and Eighth)

Grade	Problem	Omitted Word	Point of Speech
6	1	Each Containing	Subordinate clause
6	2	Modern	Adjective
6	2	If	Subordinate Conjunction
6	2	If	Subordinate Conjunction
6	2	Sleeping	Adjective
6	6	Since they started their trip	Adverbial Clause
6	11	In all	Prepositional phrase
6	14	Susies class took a field trip to the zoo	Complete Sentence
6	14	Miss Rowe Collected the money	Complete Sentence
6	14	To the Zoo	Prepositional phrase
6	14	Miss Rowe collected the money	Complete sentence
6	15	If	Subordinate Conjunction
8	7	Eats Dinner	Verb/direct object
8	12	About	Prepostion
8	12	About	Preposition

TABLE V

Completed Format Analysis of the Twelve Identified Variables in the 90 Word Problem Rewrites

Format Variables	Percentage of* Problems Involving Format Change	Percentage of** Change Made By Sixth Graders	Percentage of Change Made By Eighth Graders
1. Simplified Vocabulary	71%	40%	31%
2. Made a Long Sentence a Series of Shorter Ones	71%	34%	37%
3. Added Diagram	11%	9%	2%
4. Removed Extraneous Information	40%	23%	17%
5. Took Out "If"	47%	30%	17%
6. Took Out "About"	100%	50%	50%
7. Made Questions Sentence Separate	52%	36%	16%
8. Added Information or Changed Story Line	29%	17%	12%
9. Changed Order of Numerical Information	41%	16%	25%
10. Gave Clue	12%	8%	4%
11. Changed Verb to Present Tense	11%	9%	2%
12. Change Noun/Pronoun to Proper Noun	28%	6%	22%

*Refer to Table I. The percentages were obtained by dividing the total number of subjects making the format change by the total possible time the change could have been made in the corresponding column.

**The percentage was obtained by dividing the number of 6th graders making the format change by the total number of opportunities for the 6th graders to make the change.

TABLE VI

Analysis of Problem Type

Problem Number in Study	Computer Problem Number	Type	Operations	Length	Verbal Cue	Order	Add	Subtract	Algebra
1.	129	16	1	44	0	0	0	0	0
2.	196	8	1	26	1	0	0	0	0
3.	222	3	1	13	1	0	1	0	1
4.	225	5	1	8	1	0	0	1	1
5.	235	7	1	12	1	0	0	0	1
6.	239	30	1	28	0	1	0	1	0
7.	243	74	2	30	0	0	1	1	0
8.	253	22	1	35	1	1	0	1	0
9.	259	66	2	44	1	0	1	1	0
10.	261	76	2	23	0	0	1	0	0
11.	314	24	1	24	1	1	0	0	0
12.	345	68	2	18	1	0	1	0	0
13.	390	70	2	22	1	0	0	1	0
14.	413	80	2	54	0	0	0	0	0
15.	472	72	2	48	1	0	0	0	0

TABLE VII

DEFINITION OF VARIABLES

Variable	Name	Range	Definition
X_1	OPERS	1-4	Number of different arithmetic operations required to reach a solution, using the coded solution string.
X_2	STEPS	1-9	Number of binary operations required to obtain an answer, using the coded solution string.
X_3	LENGT	7-79	Number of words in the problem. Each number symbol (#) counts as one word. (Extraneous Information) Syntax
X_4	VCLUE	0-1	Problem has a verbal clue (coded 0) if (1) operation is + and problem has word "together" or "altogether", of if (2) operation is - and problem has phrase "have left" or "were left", or if (3) operation is x and problem has word "each."
X_5	ORDER	0-3	The number of adjacent pairs of letters in the solution string that are not in alphabetical order.
X_6	ADD	0-1	Solution requires an addition.
X_7	SUB	0-1	Solution requires a subtraction.
X_8	MUL	0-1	Solution requires a multiplication.
X_9	DIV	0-1	Solution requires a division.
X_{10}	ALGER	0-1	Problem statement is an algebraic statement, not a "story" (coded 1).

REFERENCES

Searle, B.W., Lorton, P. Jr. and Suppes, P., "Structural Variables Affecting CAI Performance on Arithmetic Word Problems of Disadvantaged and Deaf Students," Educational Studies in Mathematics, 5, (1974), pp. 371-384.

APPENDIX

Type 16

1. (129.) Over 26 billion galaxies each containing about 13 billion stars have been detected. If the average number of planets per star is 9, and the ratio of life-bearing to non-life-bearing planets is 1 to 6, how many planets (in billions) may contain another form of life?

Type 3

3. (222.) What number results from adding 26, 3715, 7, and 289, to 21?

Type 5

4. (225.) 356 is how much greater than 291?

Type 7

5. (235.) Find the number that is twice the product of 73 and 21.

6. (239.) The Kant family has driven 270 miles since they started their trip. The whole trip is 583 miles long. How many miles do they have left to go?

Type 74

7. (243.) A student does 1/3 of his homework, eats dinner, then does 1/5 more of his assignments before going to a basketball game. What part of his work was left undone?

Type 22

8. (253.) Mr. Carpenter is building a house. He uses 328 nails for the walls. Then he uses 287 nails for the roof. If he had 2913 nails to start with, how many does he have now?

Type 66

9. (259.) A football team had the ball on its own 25 yard line. On three successive plans the team made a gain of 21 yards, a loss of 7 yards, and then a gain of 8 yards. On what yard line was the football then?

Type 76

10. (261.) Scott filled 26 boxes with 6 books in each box. He had 5 extra books. How many books did he have in all?

11. (314.) The circus visited 9 cities. In all, 192,285 people saw the circus. On the average, how many people saw the circus in each city?

12. (345.) Judy shared her 35 cookies with 4 friends and herself. About how many cookies did each girl get?

13. (390.) A group of 56 people was separated into three smaller groups. Two of the smaller groups had the same number of people. The other smaller group had 10 people. How many people did each of the first two small groups have?

14. (413.) Susie's class took a field trip to the Zoo. Each student brought $2 to cover lunch, admission to the Zoo, and souvenirs. Miss Rowe collected all the money. She said that lunch cost three-fourths of all the money collected. If there were 26 people in all, how much did the group spend for lunch?

15. (472.) Water from one fish tank is carefully poured into another rectangular tank. If the water was one foot deep in the first tank, which is 4 feet long and 3 feet wide, how deep will it be in the second tank which is 3 feet by 2 feet?